

# Decay of $\rho$ and $a_1$ mesons on the lattice using distillation

**Sasa Prelovsek\***

*Jozef Stefan Institute and Department of Physics at University of Ljubljana, Ljubljana, Slovenia*

*E-mail: sasa.prelovsek@ijs.si*

**C. B. Lang**

*Institut für Physik, FB Theoretische Physik, Universität Graz, A-8010 Graz, Austria*

*E-mail: christian.lang@uni-graz.at*

**Daniel Mohler**

*TRIUMF, 4004 Wesbrook Mall Vancouver, BC V6T 2A3, Canada*

*E-mail: mohler@triumf.ca*

**Matija Vidmar**

*Jozef Stefan Institute, Ljubljana, Slovenia*

We extract the P-wave  $\pi\pi$  phase shift for five values of pion relative momenta, which gives information on the  $\rho$  resonance. The Breit-Wigner formula describes the  $\pi\pi$  phase shift dependence nicely and we extract  $m_\rho = 792(7)(8)$  MeV and the coupling  $g_{\rho\pi\pi} = 5.13(20)$  at our  $m_\pi = 266$  MeV. We extract the P-wave scattering length  $a_{l=1}^{\pi\pi} = 0.082(10)(3)$  fm<sup>3</sup> from the state with the lowest pion relative momenta.

We also determine the S-wave  $\rho\pi$  phase shift for two values of relative momenta, which provides parameters of the lowest axial resonance  $a_1(1260)$ . Using the Breit-Wigner fit we extract  $m_{a_1} = 1.44(4)$  GeV and the coupling  $g_{a_1\rho\pi} = 1.1(3)$  GeV. From the lowest state we also extract the  $\rho\pi$  scattering length  $a_{l=0}^{\rho\pi} = 0.23(12)$  fm for our  $m_\pi$ .

The simulation is performed using one  $N_f = 2$  ensemble of gauge configurations with clover-improved Wilson quarks. The phase shifts are determined from the lowest two energy-levels, which are obtained by the variational analysis with a number of quark-antiquark and meson-meson interpolators. The correlation functions are calculated using the distillation method with the Laplacian Heaviside (LapH) smearing of quarks.

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\*Speaker.

## 1. Introduction

Extracting the width of a hadronic resonance  $R$  from lattice QCD is challenging. The only proper method used up to now applies to resonances  $R$  that appear in the elastic scattering of two hadrons  $H_1 H_2 \rightarrow R \rightarrow H_1 H_2$ . First the elastic phase shift  $\delta(s)$  for  $H_1 H_2$  scattering has to be determined from the lattice for several values of  $s = E_{CM}^2 = E^2 - \mathbf{P}^2$ , where  $E$  and  $\mathbf{P}$  are the energy and the total momentum of the  $H_1 H_2$  system. Lüscher has shown that the energy  $E$  of two hadrons in a box of size  $L \simeq \text{few fm}$  provides the value of the infinite-volume elastic phase shift  $\delta(s)$  at  $s = E^2 - \mathbf{P}^2$  [1]. His relation between  $\delta$  and  $E$  for  $\mathbf{P} = 0$  was generalized to  $\mathbf{P} \neq 0$  in [2, 3, 4]. In practice, one or two lowest energy levels  $E$  are extracted and a few choices of  $\mathbf{P}$  are used in order to extract  $\delta(s)$  at different values of  $s = E^2 - \mathbf{P}^2$ .

The resulting  $\delta(s)$  can be fit with a Breit-Wigner (or any other desired) form, where both are related via the scattering amplitude  $a_l$  for the  $l$ -th partial wave

$$a_l = \frac{-\sqrt{s}\Gamma_R(s)}{s - m_R^2 + i\sqrt{s}\Gamma_R(s)} = \frac{e^{2i\delta(s)} - 1}{2i} \quad \text{or} \quad \sqrt{s}\Gamma_R(s) \cot \delta(s) = m_R^2 - s, \quad \Gamma_R(s) \propto g_{RH_1H_2}^2 \frac{p^{*2l+1}}{s} \quad (1.1)$$

and  $p^*$  is the momentum of  $H_1$  and  $H_2$  in their center-of-momentum (CMF) frame. This relation can be used to extract the mass  $m_R$  and the width  $\Gamma_R = \Gamma_R(m_R^2)$  of the resonance from lattice data on  $\delta(s)$ . The width depends significantly on the phase space and therefore on  $m_\pi$ , so it is common to extract the coupling  $g_{RH_1H_2}$ , which is expected to depend only mildly on  $m_\pi$ .

Among all the meson resonances only the  $\rho$  meson width has been determined properly using this method. The first lattice determination was done by PACS-CS in 2007 [5]. Since then, several studies of the  $\rho$  have been carried out [6, 7], with the most recent ones [8, 9, 10]. In this talk we present our recent study of the  $\rho$  [9], which achieves the smallest statistical errors (on one ensemble only, however) on the resulting  $\delta(s)$ ,  $m_\rho$  and  $\Gamma_\rho$  due to several improvements listed below.

We also extract the S-wave  $\rho\pi$  elastic phase shift, which enables us to extract the mass  $m_{a_1}$  and the width  $\Gamma_{a_1}$  of the lowest lying axial resonance  $a_1(1260)$ . The lattice study of this resonance is especially welcome as the experimental knowledge on it is very poor: the width has a wide range  $\Gamma_{a_1}^{exp} = 250 - 600 \text{ MeV}$  [11], and none of its branching ratios have been reliably determined <sup>1</sup> [11]. To our knowledge, this is the first lattice study aimed at the  $\rho\pi$  scattering and  $\Gamma_{a_1}$ .

## 2. Lattice simulation

We use 280  $N_f = 2$  configurations with tree-level clover-improved Wilson dynamical and valence quarks, corresponding to  $m_\pi a = 0.1673(16)$  or  $m_\pi = 266(3)(3) \text{ MeV}$  [12]. The lattice spacing  $a = 0.1239(13) \text{ fm}$  was determined using the Sommer parameter  $r_0$  [9] and our  $N_L^3 \times N_T = 16^3 \times 32$  is rather small, allowing us to use the powerful but costly full distillation method [13]. We combine periodic and anti-periodic propagators in time to reduce the finite  $N_T$  effects [9].

<sup>1</sup> All final states are quoted just as "seen" in [11].



**Figure 1:** Contractions for our  $\rho$  and  $a_1$  correlators with  $\bar{q}q$  and meson – meson interpolators ( $I = 1$ ).

### 3. $\rho$ resonance and $\pi\pi$ phase shift

The details of our lattice simulation aimed at  $\pi\pi$  phase shifts and the  $\rho$  resonance have been published in [9]. In this talk, we emphasize the most important steps and results.

The  $\pi^+\pi^- \rightarrow \rho^0 \rightarrow \pi^+\pi^-$  scattering is elastic below the  $4\pi$  threshold  $\sqrt{s} < 4m_\pi$  and we can apply Lüscher's method. We determine the lowest two energy-levels of the  $\rho^0 \leftrightarrow \pi^+\pi^-$  coupled system with  $J^{PC} = 1^{--}$  and  $|I, I_3\rangle = |1, 0\rangle$  for the following cases of total momentum  $\mathbf{P}$

$\mathbf{P}$	group	irrep	decay
$\mathbf{0}$	$O_h$	$T_1^-$	$\rho_3(\mathbf{0}) \rightarrow \pi(\mathbf{e}_3)\pi(-\mathbf{e}_3)$
$\frac{2\pi}{L}\mathbf{e}_3$	$D_{4h}$	$A_2^-$	$\rho_3(\mathbf{e}_3) \rightarrow \pi(\mathbf{e}_3)\pi(\mathbf{0})$
$\frac{2\pi}{L}(\mathbf{e}_1 + \mathbf{e}_2)$	$D_{2h}$	$B_1^-$	$\rho_{1,2}(\mathbf{e}_1 + \mathbf{e}_2) \rightarrow \pi(\mathbf{e}_1 + \mathbf{e}_2)\pi(\mathbf{0})$

and all permutations in direction  $\mathbf{P}$  and  $\rho$ -polarization. We display the symmetry group, the irreducible representation and the decay mode, which applies to three cases of  $\mathbf{P}$  [2, 4, 8, 9].

Other simulations aimed at  $\Gamma_\rho$  used at most one quark-antiquark interpolator and one  $\pi\pi$  interpolator for each  $\mathbf{P}$ . We use 15 quark-antiquark interpolators  $\mathcal{O}_{i=1-5}^{s=n,m,w}$  and one  $\pi\pi$  interpolator for each  $\mathbf{P}$ , where each pion is projected to a definite momentum:

$$\begin{aligned} \mathcal{O}_{i=1,\dots,5}^s(t) &= \sum_{\mathbf{x}} \frac{1}{\sqrt{2}} [\bar{u}_s(x) \mathcal{F}_i e^{i\mathbf{P}\mathbf{x}} u_s(x) - \bar{d}_s(x) \mathcal{F}_i e^{i\mathbf{P}\mathbf{x}} d_s(x)] \quad (s = n, m, w), \\ \mathcal{O}_6^n(t) &= \frac{1}{\sqrt{2}} [\pi^+(\mathbf{p}_1)\pi^-(\mathbf{p}_2) - \pi^-(\mathbf{p}_1)\pi^+(\mathbf{p}_2)], \quad \pi^\pm(\mathbf{p}_i) = \sum_{\mathbf{x}} \bar{q}_n(x) \gamma_5 \tau^\pm e^{i\mathbf{p}_i\mathbf{x}} q_n(x). \end{aligned} \quad (3.1)$$

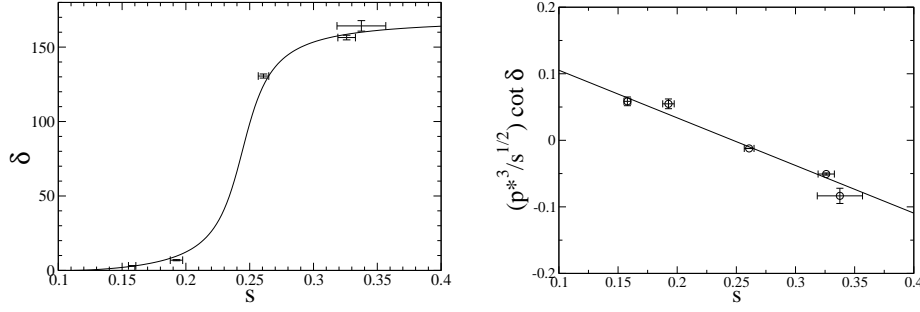
Quark-antiquark interpolators have five different color-spin-space structures  $\mathcal{F}_i$ . The quarks are smeared using the Laplacian Heaviside (LapH) smearing proposed in [13], i.e.,

$$q_s \equiv \Theta(\sigma_s^2 + \nabla^2) q = \sum_{k=1}^{N_v} \Theta(\sigma_s^2 + \lambda^{(k)}) v^{(k)} v^{(k)\dagger}, \quad s = n \text{ (narrow)}, m \text{ (middle)}, w \text{ (wide)}, \quad (3.2)$$

where different truncations  $N_v = 96, 64, 32$  correspond to three different widths  $s = n, m, w$  [9].

The  $16 \times 16$  correlation matrix  $C_{ij}(t_f, t_i) = \langle 0 | \mathcal{O}_i(t_f) \mathcal{O}_j^\dagger(t_i) | 0 \rangle$  necessitates the inclusion of the contractions in Fig. 1. The contractions were computed using the full distillation method, which is based on the LapH smeared quarks (3.2) [13] and leads to relatively precise results for all types of contractions in Fig. 1. We apply this method for the first time to extract a meson width. We also propose how to apply it for interpolators with different smearing widths in the same variational basis [9]. All correlators are expressed in terms of the so-called perambulators in Appendix A of [9]. The resulting correlators are averaged over all source time-slices  $t_i$ , over all directions of  $\mathbf{P}$  and  $\rho$  polarization.

The lowest two energies of the system are determined using the Generalized Eigenvalue Method (GEVP) [14] and the dependence on the choice of the interpolators in the variational basis is explored in [9]. The lowest energy level is robust to this choice. We find that the first excited energy



**Figure 2:** The phase shift  $\delta$  (in degrees) for  $\pi\pi$  scattering in P-wave and  $((ap^*)^3/\sqrt{sa^2})\cot\delta$  as a function of  $s$ , together with a Breit-Wigner fit.

level cannot be reliably obtained without the  $\pi\pi$  interpolator in the basis, and that more than two interpolators are required at least in the case  $P = \frac{2\pi}{L}(1, 1, 0)$ . The extracted six energy levels for our preferred interpolator choice [9] are given in Table III of [9].

Each of the six energy levels gives the value of the phase shift  $\delta(s)$  at  $s = E^2 - \mathbf{P}^2$  (<sup>2</sup>) via the Lüscher formula for  $\mathbf{P} = 0$  or its generalization to  $\mathbf{P} \neq 0$  [2, 4]. We independently confirmed the needed relations and compiled them in [9]. One of these levels,  $E_2(\mathbf{P} = 0)$ , is above the inelastic threshold  $\sqrt{s} > 4m_\pi$  and we omit it from further analysis.

The resulting phase shifts for five different values of  $s$  are plotted in Fig. 2. The phase shift has relatively small errors and exhibits a resonating behavior, which allows us to extract  $m_\rho$  and  $\Gamma_\rho$  or rather the coupling  $g_{\rho\pi\pi}$ . We use the Breit-Wigner relation (1.1) together with  $\Gamma(s) \equiv g_{\rho\pi\pi}^2 p^{*3}/(6\pi s)$ , which leads to

$$\frac{p^{*3}}{\sqrt{s}} \cot \delta(s) = \frac{6\pi}{g_{\rho\pi\pi}^2} (m_\rho^2 - s). \quad (3.3)$$

This allows a linear fit in  $s$  (Fig. 2) to extract  $m_\rho$  and  $g_{\rho\pi\pi}$  given in Table 1. The resulting  $m_\rho a = 0.4972(42)$  is slightly lower than the naive value  $m_\rho^{naive} a = 0.5107(40)$ , which is extracted from the ground state with  $\mathbf{P} = 0$ . We also extract the P-wave scattering length  $a_{l=1}^{\pi\pi} = 0.082(10)(3) \text{ fm}^3$  (defined as  $a_l \equiv \lim_{\delta \rightarrow 0} \delta(p^*)/p^{*2l+1}$  [15]) from the state with the lowest<sup>3</sup>  $p^* a = 0.1076(36)$  and  $\delta = 3.03(6)^\circ$ . This quantity is not directly measured, so we compare it to the typical value  $a_{l=1}^{\pi\pi} \simeq 0.038(2) (m_\pi^{phy})^{-3}$  obtained by combining experiment and ChPT or Roy equations [15].

A comparison of the resulting  $m_\rho$  and  $g_{\rho\pi\pi}$  to two recent lattice simulations [8, 10] is compiled in [10]. The  $N_f = 2$  simulation with twisted mass quarks [8] and the  $N_f = 2 + 1$  simulation with Wilson quarks were done at four/two values of  $m_\pi$  and explicitly demonstrate the mild dependence of  $g_{\rho\pi\pi}$  on  $m_\pi$ . All three results on  $g_{\rho\pi\pi}$  are relatively close to each other and close to the  $g_{\rho\pi\pi}^{exp} = 5.97$  extracted from  $\Gamma_\rho^{exp}$ . The resonance mass  $m_\rho$  of [10] is  $\simeq 11\%$  higher than ours, while  $m_\rho$  of [8] is  $\simeq 21\%$  higher than ours, at comparable  $m_\pi$ . Note that all three simulations get the resonance  $m_\rho$  within 3% from the value of  $m_\rho^{naive}$ , which implies that the simulations differ already in  $m_\rho^{naive}$ . Possible causes for different  $m_\rho^{naive}$  could be discretization effects or scale fixing of all

<sup>2</sup>We use the discrete dispersion relation  $\cosh(\sqrt{s}a) = \cosh(Ea) - 2\sum_{k=1}^3 \sin^2(P_k a/2)$  instead of the continuum one  $s = E^2 - \mathbf{P}^2$  to analyze the  $\rho$  [9, 5]. We analyze the  $a_1$  using the continuum dispersion relation.

<sup>3</sup>The next state leads to  $a_{l=1}^{\pi\pi}$  consistent with the value obtained from the lowest state.

	$m_\rho$ [MeV]	$g_{\rho\pi\pi}$	$a_{l=1}^{\pi\pi}$	$m_{a1}$ [GeV]	$g_{a_1\rho\pi}$ [GeV]	$a_{l=0}^{\rho\pi}$ [fm]	
latt	792(7)(8)	5.13(20)	0.082(10)(3)	1.44(4)	1.1(3)	0.23(12)	using $m_\rho$
				1.43(5)	1.7(4)	0.56(23)	using $m_\rho^{naive}$
exp	775.5	5.97	0.108(5) *	1.23(4)	< 1.35(30)	not meas.	

**Table 1:** Our lattice results for the resonance properties [9], compared to the experimental values. The results related to  $a_1$  depend on the choice of the input  $\rho$  mass:  $m_\rho$  or  $m_\rho^{naive}$ . The experimental value of  $a_{l=1}^{\pi\pi}$  is obtained combining experiment with ChPT or Roy equations.

three simulations, flavor breaking of twisted mass quarks [8] or partial quenching of the strange quark [8, 9]. Additional causes for the different  $m_\rho$  values could be the small interpolator basis in [8, 10] or the small box  $L \simeq 2$  fm in [9]. The exponentially suppressed terms, which are neglected in Lüscher formulae, may not be completely negligible for our  $L \simeq 2$  fm, which is a systematic uncertainty of our simulation. We are planning a simulation at larger  $L$  to explore possible finite size effects. We believe, however, that our small  $L$  does not influence our  $m_\rho^{naive}$ , as the first excited state  $\pi(2\pi/L)\pi(-2\pi/L)$  at  $\mathbf{P} = 0$  hardly affects the  $m_\rho^{naive}$  ground state.

Our  $\delta(s)$  agrees reasonably well with the prediction of the lowest<sup>4</sup> order of Unitarized Chiral Perturbation Theory [16], which has been recalculated for our  $m_\pi = 266$  MeV.

#### 4. The $\rho\pi$ phase shift and $a_1$ resonance

We study the S-wave scattering of  $\rho\pi$ , where the resonance  $a_1(1260)$  appears, for the total momentum  $\mathbf{P} = 0$ . The scattering is elastic at least until  $a_1(1260)$  on our lattice since  $\bar{K}^*K$  cannot be created on our  $N_f = 2$  ensemble. The ground scattering state is  $\rho(\mathbf{0})\pi(\mathbf{0})$  in the non-interacting limit. The scattering particle  $\rho(\mathbf{0})$  is almost stable on our lattice, since its lowest decay channel  $\pi(2\pi/L)\pi(-2\pi/L)$  is significantly higher in energy.

We use 9 quark-antiquark interpolators  $\mathcal{O}_{i=1-3}^{s=n,m,w}$  and one  $\rho(\mathbf{0})\pi(\mathbf{0})$  interpolator, all with  $J^{PC} = 1^{++}$ ,  $|I, I_3\rangle = |1, 0\rangle$  and  $\mathbf{P} = 0$ :

$$\mathcal{O}_1^s(t) = \sum_{\mathbf{x}, i} \frac{1}{\sqrt{2}} \bar{u}_s(x) A_i \gamma_i \gamma_5 e^{i\mathbf{P}\mathbf{x}} u_s(x) - \{u_s \leftrightarrow d_s\} \quad (s = n, m, w), \quad (4.1)$$

$$\mathcal{O}_2^s(t) = \sum_{\mathbf{x}, i, j} \frac{1}{\sqrt{2}} \bar{u}_s(x) \overleftarrow{\nabla}_j A_i \gamma_i \gamma_5 e^{i\mathbf{P}\mathbf{x}} \overrightarrow{\nabla}_j u_s(x) - \{u_s \leftrightarrow d_s\} \quad (s = n, m, w),$$

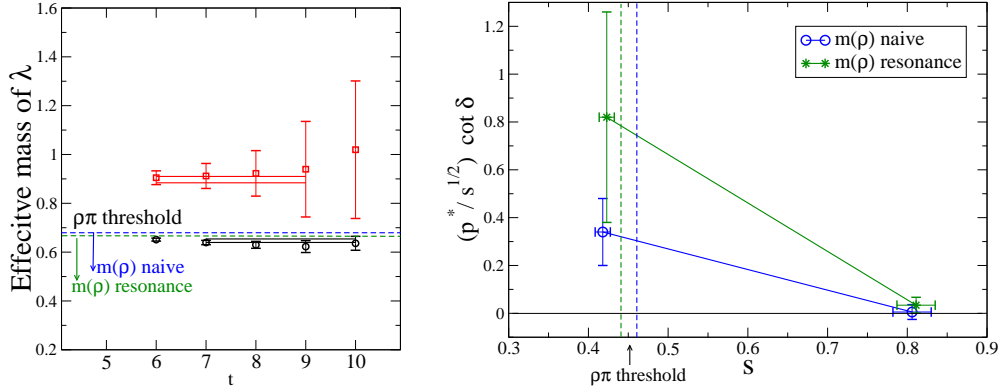
$$\mathcal{O}_3^s(t) = \sum_{\mathbf{x}, i, j, k} \frac{1}{\sqrt{2}} \varepsilon_{ijl} \bar{u}_s(x) A_i \gamma_j \frac{1}{2} [e^{i\mathbf{P}\mathbf{x}} \overrightarrow{\nabla}_l - \overleftarrow{\nabla}_l e^{i\mathbf{P}\mathbf{x}}] u_s(x) - \{u_s \leftrightarrow d_s\} \quad (s = n, m, w),$$

$$\mathcal{O}_4^n(t) = \frac{1}{\sqrt{2}} [\pi^+(\mathbf{0})\rho^-(\mathbf{0}) - \pi^-(\mathbf{0})\rho^+(\mathbf{0})], \quad \pi^\pm(\mathbf{0}) = \sum_{\mathbf{x}} \bar{q}_n \gamma_5 \tau^\pm q_n, \quad \rho^\pm(\mathbf{0}) = \sum_{\mathbf{x}} \bar{q}_n A_i \gamma_i \tau^\pm q_n,$$

where  $\nabla$  is the covariant derivative. The contractions in Fig. 1 are calculated using the full distillation method and averaged over all source time slices  $t_i$  and all  $a_1$  polarizations  $\mathbf{A}$ .

The effective mass for the lowest two eigenvalues are shown in Fig. 3 and the resulting  $E$  and  $p^*$  are given in Table 2. The CMF momentum  $p^*$  is extracted using  $E = \sqrt{p^{*2} + m_\pi^2} + \sqrt{p^{*2} + m_\rho^2}$ :

<sup>4</sup>One cannot make a fair comparison between our lattice result and the NLO prediction, since it depends on a number of LECs, and some of them have been fixed using  $m_\rho$  from another lattice study, which gets a significantly higher  $m_\rho$ .



**Figure 3:** The effective mass for lowest two eigenvalues in  $a_1$  channel (left). The combination  $p^* \cot \delta / \sqrt{s}$  as a function of  $s = E^2 - \mathbf{P}^2$ , where  $\delta$  is  $\rho\pi$  phase shift in S-wave (right).

it is imaginary for the ground state below  $m_\pi + m_\rho$  threshold, and real for the first excited state. We take two choices for the input  $\rho$  mass: our main results are based on the resonance mass  $m_\rho$  (green lines in Fig. 3), while  $m_\rho^{\text{naive}}$  is taken for comparison (blue lines in Fig. 3).

The S-wave phase shift  $\delta$  for  $\mathbf{P} = 0$  is extracted using the well known Lüscher relation [1]

$$p^* \cot \delta = \frac{2}{\sqrt{\pi} L} Z_{00} \left( 1, \left( \frac{p^* L}{2\pi} \right)^2 \right) \xrightarrow{p^* \rightarrow 0} \frac{1}{a_{l=0}^{\rho\pi}}, \quad (4.2)$$

which applies above and below threshold. The results are compiled in Table 2 for both choices of  $\rho$  mass. The first excited level gives  $\delta \approx 90^\circ$ , so it is sitting close to the top of the  $a_1$  resonance and  $m_{a_1} \approx E_2$  holds. The ground state with imaginary  $p^*$  gives imaginary  $\delta$ , but the product  $p^* \cot \delta$  is real since  $Z_{00}(1, (\frac{p^* L}{2\pi})^2)$  is real.

We parametrize  $\Gamma_{a_1}(s) \equiv g_{a_1\rho\pi}^2 p^*/s$  and apply the Breit-Wigner relation (1.1) to get

$$\frac{p^*}{\sqrt{s}} \cot \delta(s) = \frac{1}{g_{a_1\rho\pi}^2} (m_{a_1}^2 - s), \quad (4.3)$$

which applies in the vicinity of the resonance above or below threshold. Given the values of  $p^* \cot \delta$  at two different values of  $s$ , we apply a linear fit (4.3) in  $s$  (shown in Fig. 3) to extract  $m_{a_1}$  and  $g_{a_1\rho\pi}$ . The results are compiled in Table 1. Our  $m_{a_1}$  at  $m_\pi = 266$  MeV is about 14% higher than the experimental resonance  $a_1(1260)$ . The first lattice result for  $g_{a_1\rho\pi}$  is valuable, since this coupling is not known experimentally. None of the  $a_1$  branching ratios have been measured, so we provide only the upper limit for  $g_{a_1\rho\pi}^{\text{exp}}$  resulting from the total width  $\Gamma_{a_1}^{\text{exp}} = 250 - 600$  MeV. Our lattice result  $g_{a_1\rho\pi} = 1.1(3)$  GeV is in agreement with the value  $g_{a_1\rho\pi}^{\text{phen}} \approx 0.9$  GeV obtained using Unitarized Effective Field Theory approach [17] and converted to our convention. We extract also  $a_{l=0}^{\rho\pi}$  from the ground state, which is sufficiently close to the threshold. The scattering experiment cannot be carried out since  $\rho$  is a quickly decaying particle, so we compare our  $a_{l=0}^{\rho\pi}(m_\pi = 266 \text{ MeV}) = 0.23(12)$  fm to  $a_{l=0}^{\rho\pi}(m_\pi^{\text{phy}}) \approx 0.37$  fm obtained from Unitarized Effective Field Theory [18].

## 5. Conclusions

The lattice extraction of the phase shifts for elastic scattering has recently become possible also for the attractive resonant channels. We simulated the scattering in the  $\rho$  and  $a_1$  channels and

level	fit	$Ea = \sqrt{s}a$	$p^*a$	$\delta$	$p^* \cos(\delta)/\sqrt{s}$	
1	7-10	0.6468(73)	i 0.065(13)	i 7.1(54) $^\circ$	0.82(44)	(using $m_\rho$ )
			i 0.086(9)	i 23(14) $^\circ$	0.34(14)	(using $m_\rho^{naive}$ )
2	6-9	0.897(13)	0.280(10)	83.7(59) $^\circ$	0.034(33)	(using $m_\rho$ )
			0.272(10)	88.9(59) $^\circ$	0.005(31)	(using $m_\rho^{naive}$ )

**Table 2:** The results for the  $a_1 \leftrightarrow \pi\rho$  coupled channel with interpolators  $\mathcal{O}_{1,2,4}^n$  and GEVP reference time  $t_0 = 5$ . The ground state is below  $\rho\pi$  threshold, so  $p^*$  and  $\delta$  are imaginary, while the  $p^* \cot \delta$  is real.

extracted the mass and the width of these two resonances as well as the scattering lengths in the corresponding meson-meson channels.

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